Government of **Western Australia**
Curriculum Council

WACE
Western Australian Certificate of Education

Western Australian Certificate of Education Examination, 2010

Question/Answer Booklet

MATHEMATICS 3C/3D

Please place your student identification label in this box

Section Two: Calculator-assumed

Student Number: In figures | | | | | | | | |

In words _____

_____

Time allowed for this section

Reading time before commencing work: ten minutes
Working time for this section: one hundred minutes

Materials required/recommended for this section

To be provided by the supervisor

This Question/Answer Booklet
Formula Sheet (retained from Section One)

To be provided by the candidate

Standard items: pens, pencils, pencil sharpener, eraser, correction fluid/tape, ruler, highlighters

Special items: drawing instruments, templates, notes on two unfolded sheets of A4 paper, and up to three calculators satisfying the conditions set by the Curriculum Council for this course

Important note to candidates

No other items may be taken into the examination room. It is **your** responsibility to ensure that you do not have any unauthorised notes or other items of a non-personal nature in the examination room. If you have any unauthorised material with you, hand it to the supervisor **before** reading any further.

2010/31000

Ref: 10-137

Structure of this paper

Section	Number of questions available	Number of questions to be answered	Working time (minutes)	Marks available	Percentage of exam
Section One: Calculator-free	8	8	50	40	
Section Two: Calculator-assumed	12	12	100	80	
			Total	120	100

Instructions to candidates

1. The rules for the conduct of Western Australian external examinations are detailed in the *Year 12 Information Handbook 2010*. Sitting this examination implies that you agree to abide by these rules.

2. Write your answers in the spaces provided in this Question/Answer Booklet. Spare pages are included at the end of this booklet. They can be used for planning your responses and/or as additional space if required to continue an answer.
   - Planning: If you use the spare pages for planning, indicate this clearly at the top of the page.
   - Continuing an answer: If you need to use the space to continue an answer, indicate in the original answer space where the answer is continued, i.e. give the page number. Fill in the number of the question(s) that you are continuing to answer at the top of the page.

3. **Show all your working clearly.** Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat an answer to any question, ensure that you cancel the answer you do not wish to have marked.

4. It is recommended that you **do not use pencil,** except in diagrams.

See next page

Section Two: Calculator-assumed **(80 Marks)**

This section has **12** questions. Answer **all** questions. Write your answers in the space provided.

Spare pages are included at the end of this booklet. They can be used for planning your responses and/or as additional space if required to continue an answer.

- Planning: If you use the spare pages for planning, indicate this clearly at the top of the page.
- Continuing an answer: If you need to use the space to continue an answer, indicate in the original answer space where the answer is continued, i.e. give the page number. Fill in the number of the question(s) that you are continuing to answer at the top of the page.

Working time: 100 minutes.

Question 9 **(7 marks)**

Suppose that $\mathrm{P}(\mathrm{A}) = 0.5$ and that $\mathrm{P}(\mathrm{A} \cup \mathrm{B}) = 0.8$

(a) What is the maximum possible value of $\mathrm{P}(\mathrm{A} \cap \mathrm{B})$? (2 marks)

(b) What is the minimum possible value of $\mathrm{P}(\mathrm{B})$? (2 marks)

(c) What is the value of $\mathrm{P}(\mathrm{B})$ if A and B are independent? (3 marks)

Question 10 **(6 marks)**

Helium gas is being pumped into a balloon. The balloon maintains a spherical shape as it inflates, and its volume increases at a constant rate of 600 cc per minute.

(a) At what rate is the radius of the balloon increasing at the moment when the volume of the balloon is 20 litres? (1 litre = 1000 cc.) (4 marks)

(b) Use the formula $\delta y \approx \frac{dy}{dx}\delta x$ to estimate the amount by which the radius will increase in the next second. (2 marks)

See next page

Question 11 **(6 marks)**

A radioactive substance is decaying exponentially, according to the formula

$$A(t) = A(0)e^{-kt},$$ where $A(t)$ kg is the amount at time t years.

(a) Determine k, correct to 4 significant figures, given that the half-life of the substance is 12 years. (2 marks)

A second radioactive substance is also decaying exponentially, according to the formula

$$B(t) = B(0)e^{-0.04t},$$ where $B(t)$ kg is the amount at time t years.

(b) Which of these substances is decaying faster? Justify your answer briefly. (1 mark)

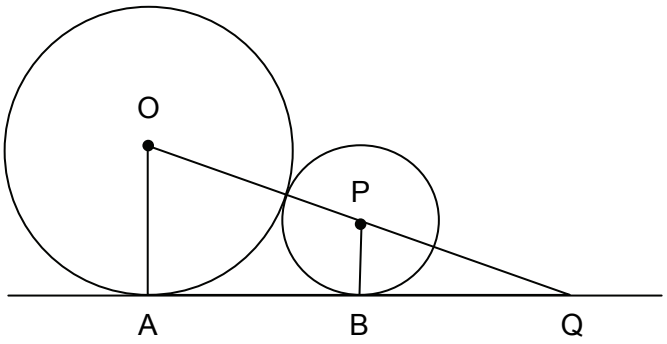
At a certain location there was exactly the same amount of these two substances at the beginning of the year 2010.

(c) In what year will the ratio of the amount of one of these substances to the other be 2:1? (3 marks)

See next page

Question 12 **(6 marks)**

Two circles are tangent to a line and to each other, as shown in the diagram below. The radius of the larger circle is twice the radius of the smaller circle.



(a) Prove that the triangles AOQ and BPQ are similar. (2 marks)

(b) Show that $PQ = 3r$ where r is the radius of the smaller circle. (2 marks)

(c) Find the radius of the smaller circle, given that AB = 20 cm. (2 marks)

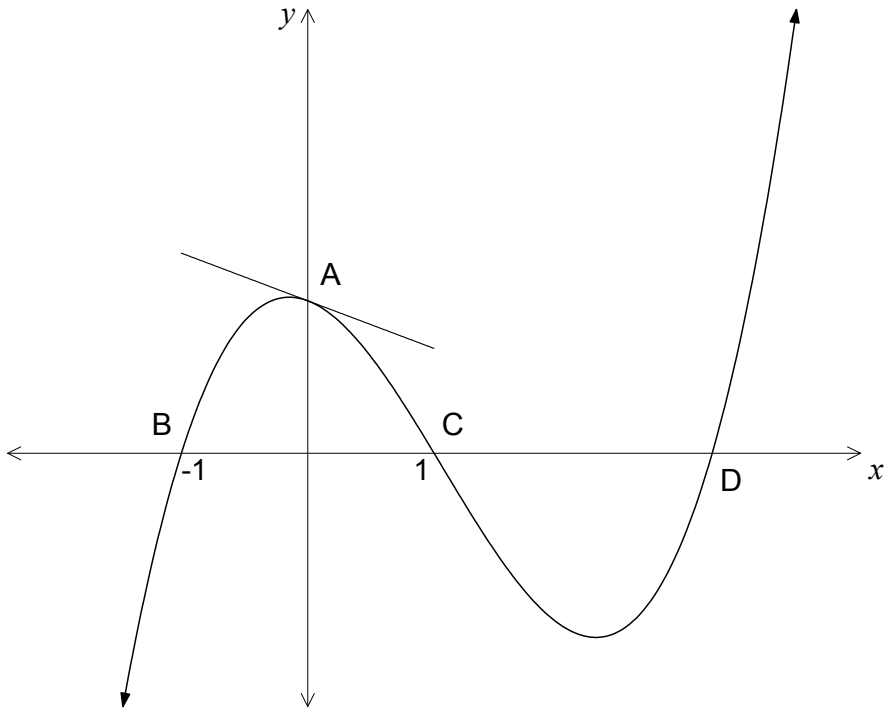
See next page

Question 13 **(5 marks)**

The diagram below shows part of the graph of $y = (x^2 - 1)(x - d)$ where $d > 1$.

The graph intercepts the *y*-axis at the point A. The graph intercepts the *x*-axis at the point B where $x = -1$, the point C where $x = 1$, and at the point D.

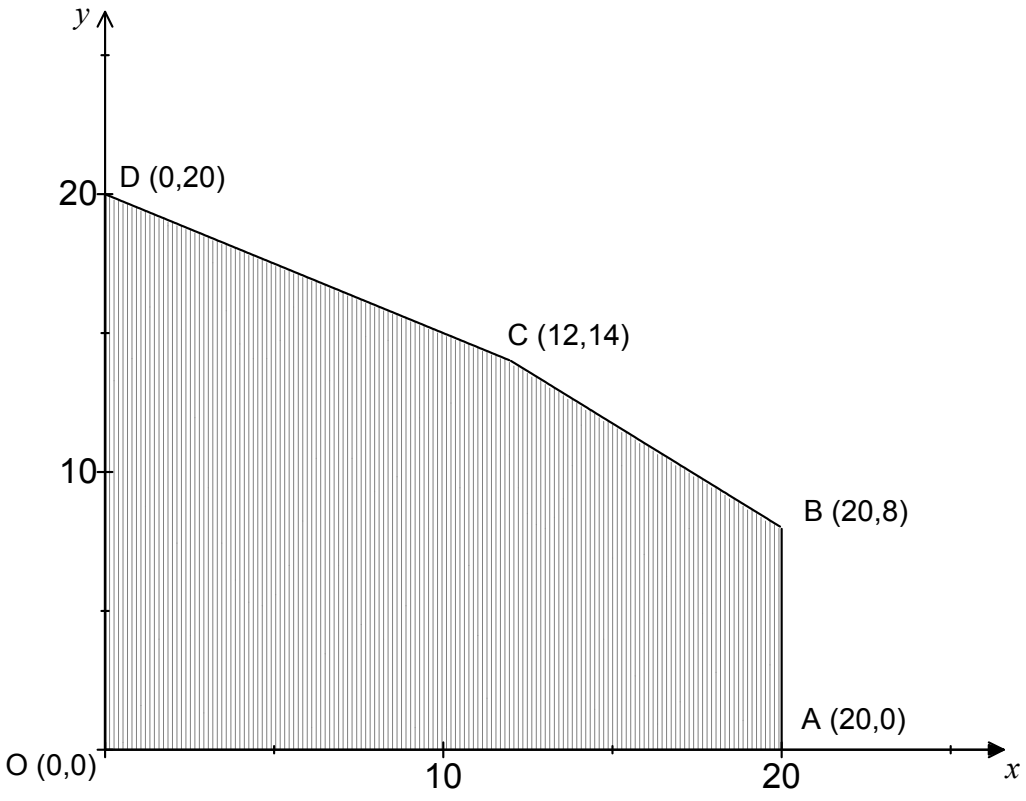
The diagram also shows part of the tangent to the graph at the point A.



Show that the tangent at A intersects the *x*-axis at D.

Question 14 **(6 marks)**

The feasible region of a linear programming problem is shown below.



The objective function is $P = 60x + 100y$

(a) Find the maximum value of P in the feasible region. (2 marks)

(b) Now suppose that the objective function changes to $P = cx + 100y$, where $c > 60$.

What is the maximum possible value of the constant c, given that the maximum value of P still occurs at the same corner point? (2 marks)

(c) Now suppose that the additional constraint $x + y \leq 27$ is imposed. Does this change the maximum value of P ? Justify your answer. (2 marks)

See next page

Question 15 **(6 marks)**

For some positive integers n the decimal expansion of $1/n$ is finite, and for others it is infinite.

For example, the decimal expansion of $1/25$ is finite since $1/25 = 0.04$, whereas the decimal expansion of $1/24$ is infinite since $1/24 = 0.41666 \cdots = 0.041\dot{6}$.

(a) Write down the decimal expansions of $1/n$ for $n = 6, 8, 11 \text{ and } 20$ (1 mark)

(b) Write down a conjecture about the prime factors of n if the decimal expansion of $1/n$ is finite.
Hint: you may wish to evaluate the decimal expansion of $1/n$, for other positive integers n, until you notice the pattern. (2 marks)

(c) Prove the claim that you made in part (b). (3 marks)

See next page

Question 16 **(5 marks)**

A new drug has been developed that can be used to test whether a person has a certain type of genetic defect. However the test is not perfect: only 95% of people with the defect have a positive reaction to the drug, and 4% of people without the defect have a positive reaction. It is also known that 2% of the total population has the genetic defect.

(a) What is the probability that a person chosen at random will have a positive reaction to the drug? (3 marks)

(b) What proportion of the people who have a positive reaction actually have the genetic defect? (2 marks)

See next page

Question 17 **(11 marks)**

The cost, $C, of insuring a house in a particular town is a uniformly distributed random variable, with minimum and maximum values $250 and $650 respectively.

The average cost is $450 and the standard deviation is $115.47.

(a) Sketch the graph of the density function of C. (2 marks)

(b) What is the probability that a randomly-chosen house in the town costs more than $500 to insure? (2 marks)

(c) What is the probability that exactly 2 of 5 randomly-chosen houses in the town cost more than $500 each to insure? (2 marks)

The total cost of insurance for 25 houses in the town owned by a real estate syndicate is $12 500. The syndicate suspects that this is unusually high.

(d) What is the average insurance cost for the houses owned by the syndicate? (1 mark)

(e) Use the Central Limit Theorem to estimate the probability that the total cost of insuring 25 randomly-chosen houses in the town will be at least $12 500. (4 marks)

See next page

Question 18 **(10 marks)**

The burn time, T seconds, of a randomly-chosen match produced by the Ever-Flame company is normally distributed, with a mean of 12.2 seconds and a standard deviation of 2.5 seconds.

(a) Calculate $P(T > 16)$ (1 mark)

(b) Find the value of *k,* given that 90% of the matches burn for longer than *k* seconds. (2 marks)

(c) If 10 matches are burned, find the probability that at least half burn for less than *k* seconds. (2 marks)

See next page

(d) Every week the company tests its matches by measuring the burn times of 1000 randomly-chosen matches.

What is the probability that the average burn time of the matches in such a sample will be between 12.15 and 12.25 seconds? (2 marks)

(e) The rival Sure-Fire company produces matches whose burn times have the same standard deviation as the Ever-Flame matches, but whose mean, μ seconds, is unknown. Scientists plan to estimate μ using the average burn time of matches in a random sample of Sure-Fire matches.

How large should this sample be, if the scientists are to be 95% confident that this estimate will be correct to within 0.1 seconds? (3 marks)

See next page

Question 19 **(7 marks)**

The acceleration, $a(t)\ \mathrm{m\ s^{-2}}$, of an object moving in a straight line is given by

$$a(t) = At + B, \text{ where } A \text{ and } B \text{ are non-zero constants.}$$

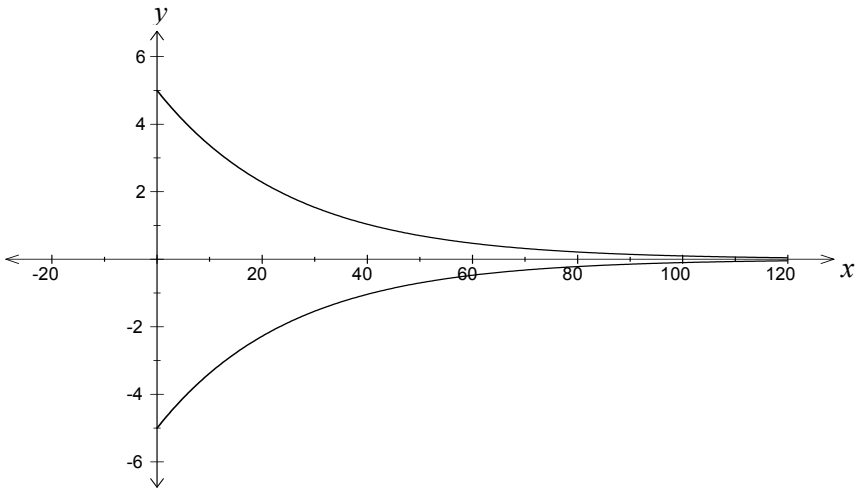
The object is at rest initially and again after 10 seconds, and the object returns to its initial position after T seconds.

(a) Evaluate T (4 marks)

(b) Evaluate A and B, given that the acceleration is positive initially and that the object travels a distance of 1 kilometre in the first T seconds. (3 marks)

Question 20 **(5 marks)**

The outline of a circularly symmetric tunnel, whose length is 120 metres, is shown below.



The top of the tunnel fits the curve $y = 5e^{-kx}$, where k is a constant, x and y are distances measured in metres, and $0 \leq x \leq 120$.
The total volume of the tunnel is 1000 cubic metres.

(a) Evaluate k correct to 3 significant figures. (3 marks)

(b) Could a man fit through the far end of the tunnel? Could a mouse? Justify your answers. (2 marks)

End of questions

Additional working space

Question number: _____

Additional working space

Question number: _____

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